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Failure Mode and Effect Analysis in a Linguistic Context: A Consensus-Based Multiattribute Group Decision-Making Approach

Hengjie Zhang , Yucheng Dong , Iván Palomares-Carrascosa, and Haiwei Zhou

Abstract—Failure mode and effect analysis (FMEA) is an effective risk-management tool, which has been extensively utilized to manage failure modes (FMs) of products, processes, systems, and services. Almost all FMEA models are concerned with how to get a complete risk order of FMs from highest to lowest risk. However, in many situations, it may be sufficient to classify the FMs into several ordinal risk classes. Meanwhile, generating a consensual decision is crucial for the FMEA problem because 1) reaching consensus will enhance the connections among FMEA participants, and 2) a highly accepted group solution to the FMEA problem can be generated. Thus, this study proposes a consensus-based group decision-making framework for FMEA with the aim of classifying FMs into several ordinal risk classes in which we assumed that FMEA participants provide their preferences in a linguistic way using possibilistic hesitant fuzzy linguistic information. In the FMEA framework, a consensus-driven methodology is presented to generate the weights of risk factors. Following this, an optimization-based consensus rule guided by a minimum adjustment distance policy is devised, and an interactive model for reaching consensus is developed to generate consensual FM risk classes. In order to justify its validity of the proposal, our framework is applied for the risk evaluation of proton beam radiotherapy.

Index Terms—Consensus, failure mode and effect analysis (FMEA), failure mode classification, multiattribute group decision-making, reliability management.

NOMENCLATURE

D	Detection.
FM	Failure mode.

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FMEA
GDM
HFLTSS
MAGDM
O
PHFLTSS
PHFLAM

RPN
S

Notations

$\{s_0, \dots, s_g\}$

H

PH

$\{TM_1, \dots, TM_m\}$

$\{FM_1, \dots, FM_n\}$

$\{RF_1, \dots, RF_y\}$

$V^{(k)}$

$V^{(c)}$

λ

Ω

C_j

T_j

$PV^{(k)}$

w

CL

FM and effect analysis.

Group decision making.

Hesitant fuzzy linguistic term sets.

Multi-attribute GDM.

Occurrence.

Possibilistic HFLTSS.

Possibilistic hesitant fuzzy linguistic assessment matrix.

Risk priority number.

Severity.

Linguistic term set.

Hesitant fuzzy linguistic term set.

Possibilistic hesitant fuzzy linguistic term set.

Set of FMEA members.

Set of FMs.

Set of risk factors.

Individual PHFLAM provided by TM_k .

Collective PHFLAM.

Weight vector of FMEA members.

Information on set of known risk factor weights.

The j th risk class of FMs.

Number of FMs in C_j .

Preference vector derived from $V^{(k)}$.

Weight vector of risk factors.

Consensus level among FMEA members.

I. INTRODUCTION

RELIABILITY engineering addresses the estimation, prevention, and management of high levels of “lifetime” engineering uncertainty and risks of failure, which has received wide attention in various areas [16], [65]. Failure mode and effects analysis (FMEA) is a highly structured, and systematic reliability-management instrument, which is very useful for evaluating and eliminating potential failure modes (FMs) of products, processes, systems, and services [5], [39], [52].

By employing FMEA tools, it is possible to identify where and how a specific product or system might fail. Likewise, the frequency, effects, and potential causes of a group of FMs can be analyzed in detail. The FMEA approach was first implemented in the United States aerospace industry by National

Aeronautics and Space Administration [4], and it has become ever since an integral tool in the Six Sigma process improvement [30]. When applying FMEA, the past design experience can be transformed into the ability to foresee future problems. In this way, some of the potential risks can be avoided at the early stages of the design. Given these advantages, FMEA has been used extensively in many industries, including aerospace, nuclear, electricity, manufacturing, marine, and healthcare [15], [23], [33], [61].

In traditional FMEA, the RPN is utilized to generate the risk priorities of potential FMs. Effectively, the RPN of an FM is determined by calculating the product of three risk factors: occurrence (O), severity (S), and detection (D). Usually, the FMs are evaluated based on each of the three risk factors (i.e., O, S, and D) using a 10-point qualitative scale [52], with the larger ordinal score indicating a stronger evidence for a hazardous situation. According to the RPN values of the identified FMs, their risk priorities are determined. Increased attention should be paid to those FMs with the highest RPN values, and a series of recommended actions is subsequently conducted to avoid or mitigate these FMs. The RPN is recalculated after the failure risk has been mitigated to confirm the effectiveness of the implemented corrective actions. For analytical references to the detailed steps on how to complete an FMEA process, we refer the interested reader to Liu [36] and Stamatis [52]. Even though the RPN-based FMEA method has been used extensively in quality improvement efforts, it has received some criticism in the literature (see [38] and [47]). First, accurate quantitative assessments on every FM with respect to risk factors are needed in the conventional RPN-based FMEA method. However, in some real-world FMEA problems, risk assessment information is often uncertain and imprecise rather than accurate, due to the lack of insightful data, time pressure, and inherent vagueness exhibited by experts in the area. Second, the conventional RPN-based FMEA methods do not take into account the relative importance of the existing risk factors, being assumed that the importance weights of risk factors are equally distributed. This assumption might be neither realistic nor precise when considering a real-world application of FMEA.

To overcome the inherent deficiencies analyzed above associated with the conventional RPN-based FMEA methods, a large body of research has been devoted in the past decades to develop and introduce various new risk priority models in the literature, most of which have focused on the effective handling of the uncertainty and imprecision in decision information at hand. For instance, Bowles and Peláez [4] initially presented a fuzzy logic-based FMEA method for dealing with some of the drawbacks in the traditional methods based on strictly numerical evaluations. Bradley and Guerrero [5] developed a method to rank FMs using a data-elicitation technique. Liu *et al.* [39] proposed an integrated FMEA approach for accurate risk assessment in an uncertain setting. Additionally, Liu *et al.* [38] and Spreafico *et al.* [51] provided a comprehensive survey of the improvement of risk evaluation methods for FMEA. An overview of the improvements made on FMEA approaches is provided in Section II.

Although the conventional FMEA models as well as the improved FMEA models have undeniably proven their usefulness

in practice, there are still many issues that need to be further investigated for coping with real-world FMEA problems.

A. Decision Outcomes

Almost all FMEA models have been focused on how to generate a complete ranking of the FMs from the most prominent to the least prominent risk. In real-world FMEA, yielding a complete ranking of FMs is sometimes very time-consuming because of the large number of FMs that are being handled [7]. Furthermore, in some situations, it is not necessary to derive a complete ranking of FMs because the FMEA goal is typically to simply distinguish between the most critical FMs and the least critical ones. In some situations, we only need to classify the FMs into several ordinal risk classes, ranking the risks from the highest to the lowest (i.e., very high, high, medium, low, and very low). For example, a large number of FMs are often involved in the risk analysis in the manufacturing processes of dairy industries, because dairy industries involve many stages, including pretreatment, filling, closing, incubation, and transportation for sharp cooling. In this case, FMs ordinal classification has some merits compared to producing a complete ranking of FMs because

- 1) the ordinal classification of FMs presents them as a structure that is easy to understand and visualize;
- 2) FMs ordinal classification allows the risk analyst to quickly access or analyze them, and leads to a more efficient decision-making and action-taking process;
- 3) FMs ordinal classification is easily implementable and requires a short computational time to get the ordered classification results of FMs.

These merits of ordinal classification of FMs have been presented in Certa *et al.* [9]. To our knowledge, there are very few FMEA approaches that have focused on the ordinal classification of the FMs into ordinal classes with the exception of the approaches presented in Certa *et al.* [9] and Lolli *et al.* [41].

B. Diversity in Decision Group Opinions

FMEA team members typically come from different areas and may differ in the knowledge structure, evaluation levels, as well as practical experience, and their preferences may thus differ substantially. Most extant FMEA methods do not take this issue into account, and they only focus on how to obtain a ranking of the FMs by fusing FMEA team member preference information without addressing the issue on whether or not the consensus level among FMEA team members can be guaranteed. In practice, achieving a consensus among FMEA participants is a crucial aspect to consider, which offers a few key advantages, such as 1) building connections among the FMEA participants. Using a consensus-reaching model as a decision tool means taking the time to find unity on how to proceed before moving forward which promotes communication among FMEA participants. 2) A more effective and accepted implementation of the decision results. When FMEA participant preferences and concerns are taken into account, they are much more likely to actively participate in the implementation of the obtained solution to the FMEA problem. A detailed analysis of the above advantages of

the consensus-reaching model has been discussed in Susskind *et al.* [53].

To deal with a decision problem involving multiple individuals, many group decision making (GDM) and multiattribute group decision making (MAGDM) models have been reported in the literature [28]. In particular, numerous consensus-based GDM and MAGDM models have been designed for supporting reaching consensus among a group of individuals [2], [6], [10], [11], [17], [19], [58]. The comprehensive overview of related works provided in Section II demonstrates that research on consensus-reaching process has been the subject of numerous achievements [1], [46], [52]. However, they cannot be applied in the ordinal classification of FMs based on the FMEA problem in a straightforward manner because all of them are focused on obtaining a consensual complete ranking of alternatives (with FMs being deemed as alternatives in the decision-making problem) rather than producing an ordinal classification of alternatives. It is therefore crucial and necessary to develop a consensus-reaching model for supporting the achievement of consensus in the ordinal classification-based FMEA problem of FMs.

Due to the complexity of real-world decision situations, some individuals may often face difficulties to provide their opinions in a precise manner. The hesitant fuzzy set (HFS) [54], [60], [62] is an effective way for modeling uncertain opinions in decision making, and their membership functions are represented by a set of possible values. Meanwhile, decision makers will often be more comfortable in expressing their opinions in a linguistic way. Thus, by combining the merits of the HFSs and linguistic term sets, Rodríguez *et al.* [48] further proposed the concept of the HFLTS to increase the flexibility and expressiveness power of elicited linguistic preferential information. Furthermore, by incorporating the possibilistic information into the hesitant fuzzy linguistic information, the possibilistic hesitant fuzzy linguistic assessment model was developed [59]. The possibilistic hesitant fuzzy linguistic assessment model is a useful tool for FMEA members to express their uncertain assessment information due to its convenience and flexibility in handling the hesitancy and uncertainty underlying such assessments in practical contexts [57]. Therefore, it constitutes the preference modeling approach adopted in this study.

Motivated by the challenges of filling the research gaps and challenges highlighted above on the existing FMEA models, and inspired by the advances achieved on reaching consensus in the GDM, we propose a consensus-based GDM approach for FMEA problems in a possibilistic hesitant fuzzy linguistic context with the aim of classifying the system/process FMs into several ordinal risk classes (e.g., very high, high, medium, low, and very low). In the proposed consensus-based FMEA framework, we present a consensus-driven methodology to compute the weights of the risk factors in the context of incompleteness, thereby enabling a more realistic setting where not all risk factors may be equally important. Following this, we present a consensus rule founded on a minimum adjustment distance, and propose an optimization model to support this consensus rule. The optimization model is converted into a 0–1 mixed linear programming model to facilitate its resolution. We further develop an interactive consensus-reaching/building process for FMEA

problems on the basis of the proposed consensus rule. In the consensus-reaching process, FMEA team members can adjust their preferences flexibly according to the adjustment suggestions generated by the optimization-based consensus rule based on minimum adjustment distances. Finally, a case study regarding the problem of evaluating risk in proton beam radiotherapy is presented to justify the feasibility and validity of the proposed methodology.

The remainder of this study is arranged as follows. In Section II, we briefly review the literature of improved FMEA methods as well as the literature regarding consensus building in GDM. Section III introduces preliminaries regarding linguistic decision-making representational models considered in this study. Section IV presents the target consensus-based FMEA problem, proposing its resolution framework. Following this, Section V devises a consensus-driven optimization-based model to determine the weights of risk factors, and Section VI develops a consensus-reaching process with a minimum adjustment distance to support reaching consensus in the FMEA problem. Subsequently, the feasibility and validity of the proposed FMEA method are demonstrated using a case study in Section VII and a comparison analysis is completed in Section VIII. Finally, the conclusions of this study and a discussion on future research directions are outlined in Section IX.

II. LITERATURE REVIEW

This section reviews some related works on improved FMEA methods as well as the consensus-reaching processes in GDM problems owing to their relevance with the scope of this study.

A. Failure Mode Evaluations in FMEA

As mentioned previously, it is often difficult for an FMEA expert to quantify his/her assessment as an exact value in a numerical scale such as 1–10, for instance. Thus, a large number of approaches/methodologies have been reported to model the uncertainties of the assessment information from FMEA team members. Bowles and Peláez [4] introduced a fuzzy logic theory approach for generating a rankings of FMs involved in an FMEA problem. Additional FMEA approaches that are based on fuzzy logic theory can be found in [23], [31], [43], [47], [63], and [65]. An evidential reasoning approach is employed by several researchers to deal with the assessment of information with uncertainty in the FMEA, including those proposed by Chin *et al.* [12] and Liu *et al.* [37]. Adhikary *et al.* [18] adopted gray numbers to quantify the assessment information of the FMs with respect to the risk factors. In addition, the linguistic assessment approach has been utilized to deal with the uncertainty faced by FMEA team members' evaluation information. Based on interval two-tuple linguistic information, a rigorous risk ranking method was proposed by Liu *et al.* [39] to improve the FMEA accuracy. Recently, Huang *et al.* [29] applied linguistic distribution assessments to represent risk evaluation information collected from FMEA team members. Other FMEA methods have been reported to model the ambiguity involved in FMEA problems, such as those using rough sets [49], and two-tuple linguistic variables [35].

Moreover, HFLTSs [48], which increased the flexibility and richness of linguistic elicitation, have been recently applied in the FMEA by Liu *et al.* [40]. Furthermore, the possibilistic hesitant fuzzy linguistic term sets (PHFLTSs) have been developed in the literature by incorporating possibilistic information [59]. Compared with other linguistic models, the PHFLTSs are more effective in modeling the uncertainty and hesitancy in practical applications. Thus, the use of a linguistic method based on PHFLTSs provides an added value in managing linguistic risk evaluations in the FMEA problem. To our knowledge, PHFLTSs have not been adopted yet by existing FMEA models to denote uncertain assessments by FMEA members.

B. Risk Factor Weights in FMEA

To overcome the drawback of RPN-based FMEA concerning the importance of risk factors, many approaches, including subjective and objective weighting approaches, have been reported to derive the importance of risk factors. The direct assessment [39], analytic hierarchy process [64], and Delphi methods [67], are commonly used methods for the determination of the subjective importance of risk factors. Additionally, the data envelopment analysis [12], [13] is typically utilized to deduce the objective risk factor weights. Liu *et al.* [39] utilized a combined approach to compute the degrees of importance of the risk factors in FMEA, in which the objective weights were derived based on statistical distances. Song *et al.* [50] employed the entropy-based weighting approach for computing the risk factors' objective weights, and a combined approach was then presented to integrate subjective and objective weights of risk factors. Recently, Liu *et al.* [40] reported a novel weight determination method, and the basic principle of this method was based on the fact that the most serious FM(s) should have the "greatest relation grade" to the reference sequence.

When applying FMEA, a multidisciplinary team that consists of multiple experts, a group of FMs, and a set of risk factors are often involved. The team members express their assessment information of FMs with respect to multiple risk factors. Thus, the FMEA can be regarded as a complicated MAGDM problem [40]. In particular, the FMEA team members can be seen as decision makers, and FMs can be deemed as alternatives, while risk factors can be perceived as attributes in the MAGDM. In MAGDM problems, consensus-driven approaches aimed at maximizing the consensus level among all individuals have been recently adopted with the additional aim of determining the weights of attributes [20]. To our knowledge, there is no research focused on undertaking a consensus-driven approach for computing the weights of risk factors in FMEA problems as of yet.

C. Prioritization of FMs in FMEA

As mentioned above, the determination of the priority ordering of FMs in FMEA can be seen as an MAGDM problem [40]. It is worth noting that the MADM or MAGDM methods have proved to be useful approach to rank FMs.

For instance, Franceschini and Galetto [22] presented an MADM method for determining the risk order of FMs in FMEA.

Their method is capable of addressing qualitative assessment information without necessitating a numerical conversion. Song *et al.* [50] adopted a TOPSIS method to produce the priority ordering of FMs in FMEA. Liu *et al.* [40] reported an integrated MADM model to generate the risk order of FMs under the context of uncertainty. Huan *et al.* [29] applied an improved TODIM-based FMEA method for determining the risk order of FMs. Mohsen and Fereshteh [42] proposed an extended Visekriterijumska optimizacija i Kompromisno Resenje (VIKOR) method based on an entropy measure for the FM risk assessment. Wang *et al.* [55] reported an FMEA method by using the house of reliability-based rough VIKOR approach. Other studies regarding prioritization of FMs can be found in [5], [12], [13], [18], [23], and [31].

The above prioritization approaches are all focused on how to generate a complete ranking of FMs rather than on the ordinal classification of FMs. However, deriving a complete ranking of FMs is sometimes infeasible in practice owing to a possibly large number of FMs that are being handled [7]. Furthermore, some real-life scenarios, such as FMEA do not require a complete ranking of FMs [9], [41]. Certa *et al.* [9] developed an alternative approach for the criticality assessment of process/system FMs. In their work, the ELECTRE TRI method was utilized to classify FMs into several ordinal risk classes with the risk levels ranked from high to low. Lolli *et al.* [41] developed an MADM method named FlowSort-GDSS to divide FMs into several ordinal risk classes. In addition, both the method introduced by Lolli *et al.* and all other existing FMEA models focused on the direct aggregation of the different assessments of FMEA member information and the prioritization of FMs, in such a manner so that the consensus was not addressed among FMEA members.

D. Consensus-Reaching Processes in the GDM

A vast number of consensus models have been reported in literature to help decision makers reach a consensus in GDM. For example, Altuzarra *et al.* [2] investigated the problem of reaching consensus in AHP-GDM from a Bayesian perspective. Herrera-Viedma *et al.* [25] and Choudhury *et al.* [14] proposed consensus models for GDM problems with different preference representation structures. Ben-Arieh *et al.* [3] devised a minimum cost consensus with quadratic cost functions. Wu and Xu [57] developed consensus frameworks that simultaneously managed individual consistency and consensus in GDM with hesitant fuzzy linguistic preference relations. Pérez *et al.* [44] suggested a dynamic consensus model to manage decision situations in which the set of alternatives changed dynamically. Moreover, Alonso *et al.* [1] and Kacprzyk and Zadrożny [32] developed web-based consensus support systems. Recently, Capuano *et al.* [8] and Wu *et al.* [56] developed two approaches for undertaking consensus-reaching processes in which the trust relationship among individuals was considered. Palomares *et al.* [46] presented a consensus model for large-group decision making capable of identifying and managing noncooperative behaviors. Additional approaches for establishing consensus in GDM can be found in Dong *et al.* [19] and Herrera-Viedma *et al.* [24]. A detailed survey of existing consensus models un-

der fuzzy contexts can be found in Palomares *et al.* [45]. To our knowledge, all these consensus models were focused on how to generate a complete ranking of alternatives under consensus rather than deriving a classification of decision alternatives on several ordinal classes.

From the above literature, the following observations are outlined:

- 1) the PHFLTSs have not been adopted to model uncertain assessment information by existing FMEA models;
- 2) most FMEA models are focused on how to generate a complete risk order of FMs from the highest to the lowest risk, and there is a clear shortage of FMEA approaches that focus on the ordinal classification of the FMs;
- 3) the consensus among all FMEA team members is not taken into account by any of the existing FMEA approaches, and the existing consensus-reaching processes are envisaged to determine a comprehensive ranking of alternatives.

Therefore, they cannot be applied directly in the FM ordinal classification-based FMEA problem. All of these research gaps motivated us to propose a consensus-based MAGDM approach for the FMs ordinal classification-based FMEA problem in the possibilistic hesitant fuzzy linguistic context.

III. PRELIMINARIES

This section introduces some basic knowledge regarding the two-tuple linguistic model, and the possibilistic hesitant fuzzy linguistic assessments information, which constitute the representative models for linguistic preferential information utilized in the framework proposed in this study.

A. Two-Tuple Linguistic Model

The basic notations and operational laws of linguistic variables were introduced in Herrera and Martínez [27]. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set with odd granularity $g + 1$, where the term s_j signifies a possible value for a linguistic variable. The linguistic term set is typically required to satisfy the following additional characteristics:

- 1) the set is ordered: $s_i \leq s_j$ if and only if $i \leq j$;
- 2) there is a negation operator such that $\text{neg}(s_j) = s_{g-j}$.

Herrera and Martínez [27] reported a notable symbolic model for computation with words: the two-tuple linguistic model. Let S be a linguistic term set within the granularity interval $[0, g]$. The two-tuple that expresses the equivalent information to $\beta \in [0, g]$ can be obtained using the following function:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5], \quad (1)$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5]. \end{cases} \quad (2)$$

In the Herrera and Martínez model, Δ represents a one-to-one mapping function. For convenience, its range is denoted as \bar{S} . The function Δ has an inverse function $\Delta^{-1} : \bar{S} \rightarrow [0, g]$ with $\Delta^{-1}((s_i, \alpha)) = i + \alpha$. For notation simplicity, this study set $\Delta^{-1}((s_i, 0)) = \Delta^{-1}((s_i))$.

Let (s_i, α) and (s_j, γ) be two linguistic two-tuples. If $\Delta^{-1}((s_i, \alpha)) < \Delta^{-1}((s_j, \gamma))$, then (s_i, α) is smaller than (s_j, γ) .

The improvements of the two-tuple linguistic model have also been developed, such as the model with a hierarchical structure [26], and the numerical scale model [21].

B. Possibilistic Hesitant Fuzzy Linguistic Assessment Information

The aforementioned two-tuple linguistic model is useful to address the linguistic decision-making problems with a single linguistic term. However, similar to HFSs [54], in linguistic setting, decision makers may hesitate to choose values among several available values when assessing a linguistic variable. To deal with these situations, Rodríguez *et al.* [48] proposed the concept of HFLTS to increase the richness and flexibility of elicited linguistic information.

Definition 1: [48]. Let $S = \{s_0, \dots, s_g\}$ be a predefined linguistic term set. Let L and U be two inters, where $L, U \in \{0, 1, \dots, g\}$ and $L \leq U$. The HFLTS, $H = \{s_L, s_{L+1}, \dots, s_U\}$, is thus an ordered finite subset of consecutive linguistic terms of S , where s_L and s_U are the lower and upper bounds of H , respectively.

By incorporating possibilistic information into HFLTSs, PHFLTSs have been developed [59].

Definition 2: Let $S = \{s_0, \dots, s_g\}$ be as defined above. A PHFLTS is denoted by $\text{PH} = \{(s_L, p_L), (s_{L+1}, p_{L+1}), \dots, (s_U, p_U)\}$, where s_L and s_U are the lower and upper bounds of PH, respectively, and $p_i \in [0, 1]$ denote the possibility degree of linguistic term s_i and $\sum_{i=L}^U p_i = 1$.

For convenience, we use M^S to denote a set of PHFLTSs based on S .

Let PH be the mean (or expected value) for PH that can be calculated in the following manner:

$$E(\text{PH}) = \Delta \left(\sum_{i=L}^U \Delta^{-1}(s_i) \cdot p_i \right). \quad (3)$$

Clearly, $E(\text{PH}) \in \bar{S}$. Let $\text{PH}_i = \{(s_{L(i)}, p_{L(i)}^{(i)}), (s_{L(i)+1}, p_{L(i)+1}^{(i)}), \dots, (s_{U(i)}, p_{U(i)}^{(i)})\}$ and $\text{PH}_j = \{(s_{L(j)}, p_{L(j)}^{(j)}), (s_{L(j)+1}, p_{L(j)+1}^{(j)}), \dots, (s_{U(j)}, p_{U(j)}^{(j)})\}$ be two PHFLTSs. The comparison operation over PH_i and PH_j can be defined as follows: if $E(\text{PH}_i) < E(\text{PH}_j)$, then $\text{PH}_i < \text{PH}_j$, if $E(\text{PH}_i) = E(\text{PH}_j)$, then $\text{PH}_i = \text{PH}_j$.

Definition 3: Let PH_i and PH_j be defined as above. The distance between PH_i and PH_j is defined by the following:

$$d(\text{PH}_i, \text{PH}_j) = \frac{|\Delta^{-1}(E(\text{PH}_i)) - \Delta^{-1}(E(\text{PH}_j))|}{g}. \quad (4)$$

Clearly, $d(\text{PH}_i, \text{PH}_j) \in [0, 1]$. A larger value of $d(\text{PH}_i, \text{PH}_j)$ indicates a larger deviation between PH_i and PH_j .

Definition 4: Let $H = \{s_L, s_{L+1}, \dots, s_U\}$ be as defined above. Correspondingly, H can then be transformed into a PHFLTS,

$PH = \{(s_L, p_L), (s_{L+1}, p_{L+1}), \dots, (s_U, p_U)\}$, where

$$p_i = \frac{1}{U - L + 1}, i = L, L+1, \dots, U. \quad (5)$$

Definition 5: Let $PH = \{(s_L, p_L), (s_{L+1}, p_{L+1}), \dots, (s_U, p_U)\}$ be as defined above. Thus, PH can be transformed into a PHFLTS over all linguistic terms in S , $\overline{PH} = \{(s_i, \hat{p}_i) | i = 0, 1, \dots, g\}$, where

$$\hat{p}_i = \begin{cases} p_i, & i = L, L+1, \dots, U \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Example 1: Let $PH = \{(s_2, 0.2), (s_3, 0.3), (s_4, 0.5)\}$ be a PHFLTS defined on $S = \{s_0, \dots, s_6\}$. Using (6), PH can be converted into $\overline{PH} = \{(s_0, 0), (s_1, 0), (s_2, 0.2), (s_3, 0.3), (s_4, 0.5), (s_5, 0), (s_6, 0)\}$.

Let $\{PH_1, \dots, PH_n\}$ be a set of PHFLTSs, where $PH_k = \{(s_{L(k)}, p_{L(k)}^k), (s_{L(k)+1}, p_{L(k)+1}^k), \dots, (s_{U(k)}, p_{U(k)}^k)\}$. Let $\pi = (\pi_1, \pi_2, \dots, \pi_n)^T$ be a weight vector that satisfies $0 \leq \pi_i \leq 1$ and $\sum_{k=1}^n \pi_k = 1$. The collective PHFLT over all linguistic terms in S , $PH^{(c)} = \{(s_i, p_i^{(c)}) | i = 0, 1, \dots, g\}$, can be generated using the hesitant fuzzy linguistic weighted average (HFLWA) operator [59], that is

$$(s_i, p_i^{(c)}) = \text{HFLWA}_\pi(PH_1, \dots, PH_n) = (s_i, \sum_{k=1}^n \pi_k \cdot \hat{p}_i^{(k)}) \quad (7)$$

where $\hat{p}_i^{(k)}$ is derived from PH_k using (6).

IV. CONSENSUS-BASED FMEA PROBLEM AND ITS RESOLUTION FRAMEWORK

In this section, we formally present the target consensus-based FMEA problem in our study, and we design an MAGDM framework to facilitate its resolution.

A. Presentation of the Consensus-Based FMEA Problem

In an FMEA problem, human decision-making behaviors are inherently subjective to a certain extent. For this reason, it becomes reasonable to collect the assessment information on the risks of FMs using a linguistic assessment domain. Possibilistic hesitant fuzzy linguistic information can efficiently convey the linguistic judgments of individuals. Thus, this study uses the possibilistic hesitant fuzzy linguistic approach to address linguistic assessment information from FMEA team members. As mentioned in Section II, there are many practical FMEA situations in which the sole aim is to classify the FMs into several ordinal risk classes. Meanwhile, making consensual decisions is a paramount aspect in FMEA problems: the aim of reaching consensus in such contexts is to assist FMEA team members in improving the consensus level so as to identify acceptable collective ordinal risk classes of FMs to the FMEA problem at hand.

Herein, we propose the consensus-based FMEA problem with the aim of classifying FMs into several ordinal risk classes, which is formally proposed as follows. Suppose that there are m team members $TM = \{TM_1, TM_2, \dots, TM_m\}$ in an FMEA, and they need to provide assessment information of a group

of n potential FMs $FM = \{FM_1, FM_2, \dots, FM_n\}$ against a group of y risk factors $RF = \{RF_1, RF_2, \dots, RF_y\}$. In this study, the FMEA team members provide their assessment information on FMs against each risk factor using the possibilistic hesitant fuzzy linguistic approach. Let $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ be a PHFLAM given by a team member $TM_k \in TM$, where $v_{ij}^{(k)} = \{(s_{L(ij)}^{(k)}, p_{L(ij)}^{(k)}), (s_{L(ij)+1}^{(k)}, p_{L(ij)+1}^{(k)}), \dots, (s_{U(ij)}^{(k)}, p_{U(ij)}^{(k)})\} \in M^S$ represents the possibilistic hesitant fuzzy linguistic assessment of FM FM_i over risk factor RF_j . Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$ be the weight vector of FMEA members, where $\lambda_k \in [0, 1]$ signifies the relative weight of TM_k , thus satisfying $\sum_{k=1}^m \lambda_k = 1$. Several methods have been reported to calculate $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$ (see [12], [55]). The weight vector of risk factors is denoted by $w = (w_1, w_2, \dots, w_y)^T$, where $w_i \geq 0$ ($i = 1, 2, \dots, y$) denotes the relative weight of RF_i , satisfying $\sum_{i=1}^y w_i = 1$. In this study, the weights of risk factors are considered as partially known.

The problem in this study is concerned with finding a consensual classification of FMs into several ordinal classes using the individually elicited PHFLAMs $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ ($k = 1, 2, \dots, m$). Without loss of generality, the FMs $FM = \{FM_1, FM_2, \dots, FM_n\}$ are needed to classify them into q ($q \geq 2$) ordinal risk classes, which are denoted as C_1, C_2, \dots, C_q , respectively. The risk degree of FM in C_i is larger than that in C_j if $i < j$, and the number of FMs in C_i is denoted as T_i .

B. Resolution Framework

As mentioned in Section II, all the FMEA approaches except the works of Certa *et al.* [9] and Lolli *et al.* [41] have focused on how to generate the complete risk order of FMs from the highest to the lowest risk. The weights of risk factors are considered as partially known in this paper. Inspired by recent consensus models with minimum adjustment distance [3], [19], [25], we propose a consensus-based FMEA framework with the aim of classifying the FMs into several ordinal risk classes, as presented in Fig. 1.

In this framework, there are two key processes.

1) *Application of Consensus-Driven Methodology to Generate the Weights of Risk Factors:* In this step, the weights of risk factors are determined by minimizing the degree of divergence among all FMEA team members. Meanwhile, the consensus level among FMEA members, and the individual and collective ordinal risk classes of FMs are also yielded.

If the consensus level among the FMEA members is acceptable, then the risk analysis process is completed. Otherwise, the consensus-reaching process is undertaken to help FMEA members modify their possibilistic hesitant fuzzy linguistic assessments on FMs to improve the consensus level regarding the obtained collective ordinal risk classes of FMs.

The details of this process are presented in Section V.

2) *Consensus-Reaching Process:* In the consensus-reaching process, an optimization-based model is designed to help FMEA members obtain their optimally adjusted linguistic assessment information, which are used as the references for FMEA members to modify their linguistic assessment information.

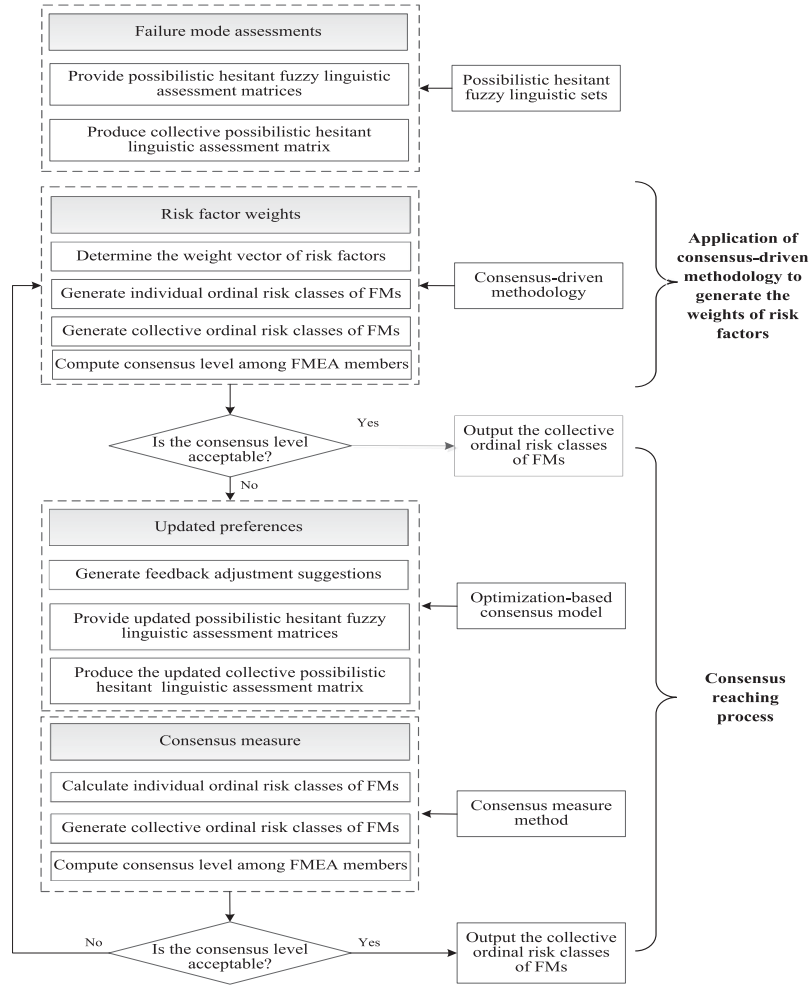


Fig. 1. Consensus-based FMEA framework.

The detailed information of the consensus-reaching process are presented in Section VI.

This framework is described as follows: After the FMEA members provide their possibilistic hesitant fuzzy linguistic assessment information on FMs in the form of PHFLAMs, these PHFLAMs are then aggregated into a collective form. Following this, a consensus-driven methodology is presented to generate the weights of risk factors from the information on incomplete weights. Meanwhile, according to the constructed consensus-driven optimization model, the consensus degree among all FMEA members, the individual and collective ordinal risk classes of FMs can also be obtained. If the current consensus level among the FMEA members is acceptable, the consensus-reaching process is terminated. Otherwise, an optimization-based consensus building mechanism with minimum adjustment distance is constructed to help FMEA members improve their consensus level. This procedure is followed until the predefined consensus level among all FMEA members is reached.

V. CONSENSUS-DRIVEN OPTIMIZATION-BASED MODEL TO DETERMINE THE WEIGHTS OF RISK FACTORS

In this section, we describe the procedure for determining the importance weights of risk factors from the incomplete weights information using a consensus-driven methodology.

A. Format of the Weights of the Risk Factors

Let $w = (w_1, w_2, \dots, w_y)^T$ be expressed as above. The known weight information on the risk factor RF_j ($j = 1, 2, \dots, y$) can be typically constructed using the following basic forms [34], for $i \neq j$:

- 1) weak ranking: $\Omega_1 = \{w_i \geq w_j\}$;
- 2) strict ranking: $\Omega_2 = \{w_i - w_j \geq \gamma_{ij}\} \ (\gamma_{ij} > 0)$;
- 3) ranking of differences: $\Omega_3 = \{w_i - w_j \geq w_k - w_t\} \ (j \neq k \neq l)$;
- 4) ranking with multiples: $\Omega_4 = \{w_i \geq \gamma_{ij} \cdot w_j\} \ (j \neq k \neq l)$;
- 5) interval form: $\Omega_5 = \{\gamma_i \leq w_i \leq \gamma_i + \varepsilon_i\} \ (0 \leq \gamma_i \leq \gamma_i + \varepsilon_i \leq 1)$.

In practical FMEA, the risk factor weight structure forms often consist of multiple basic forms as presented above. Without loss of generality, we use Ω to denote the set of known risk factor weight information provided by FMEA team members. In particular, $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5$.

Naturally, the consensus level among all the FMEA team members is expected to be as high as possible. Following this idea, we present an optimization-based model to minimize the divergence degree among all FMEA team members by optimizing the weights of risk factors. Before formally presenting the optimization model, we present several relevant concepts.

B. Generating the Individual and Collective Ordinal Risk Classes of the FMs

In this process, the individual and collective ordinal risk classifications of the FMs are generated. Two steps are included in this process: aggregation and exploitation.

1) *Aggregating FMEA Team Members' Assessments Into a Collective Assessment Matrix:* Let $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ be denoted as above. The collective PHFLAM $V^{(c)} = (v_{ij}^{(c)})_{n \times y}$ can be generated using the HFLWA operator, where $v_{ij}^{(c)} = \{(s_t, p_{ij,t}^{(c)}), t = 0, 1, \dots, g\}$ is computed as indicated as follows:

$$(s_t, p_{ij,t}^{(c)}) = \text{HFLWA}_\lambda \left(v_{ij}^{(1)}, \dots, v_{ij}^{(m)} \right) = \left(s_t, \sum_{k=1}^m \lambda_k \cdot \hat{p}_{ij,t}^{(k)} \right) \quad (8)$$

where $\hat{p}_{ij,t}^{(k)}$ is obtained from $v_{ij}^{(k)}$ using (6).

2) *Using the Exploitation Operation to Generate the Individual and Collective Ordinal Risk Classes of FMs:* In this step, the individual and collective ordinal risk classes of FMs are obtained.

Let $PV = (pv_1, pv_2, \dots, pv_n)^T$ be the preference vector derived from PHFLAM $V = (v_{ij})_{n \times y}$, where $pv_i \in [0, g]$ denotes the preference value of FM_i , and calculated by,

$$pv_i = \sum_{j=1}^y w_j \cdot \Delta^{-1}(E(v_{ij})). \quad (9)$$

For convenience, the preference vectors generated from $V^{(k)}$ and $V^{(c)}$ are denoted as $PV^{(k)}$ and $PV^{(c)}$, respectively.

Based on PV , the risk order of FMs, $RO = (ro_1, ro_2, \dots, ro_n)^T$, can be achieved, where

$$ro_i = j \quad (10)$$

if pv_i is the j th largest value in $\{pv_1, pv_2, \dots, pv_n\}$.

For convenience, the risk orders derived from $PV^{(k)}$ and $PV^{(c)}$ are denoted as $RO^{(k)} = (ro_1^{(k)}, \dots, ro_n^{(k)})^T$ and $RO^{(c)} = (ro_1^{(c)}, \dots, ro_n^{(c)})^T$, respectively.

Furthermore, according to $RO = (ro_1, \dots, ro_n)^T$, $FM = \{FM_1, \dots, FM_n\}$ can be classified into q ordinal risk classes, C_1, C_2, \dots, C_q , where

$$FM_i \in \begin{cases} C_1, & ro_i \leq T_1 \\ C_t, & T_1 + \dots + T_{t-1} + 1 \leq ro_i \leq T_1 + \dots + T_t \\ C_q, & ro_i < n - T_q \end{cases} \quad (11)$$

Obviously, $FM_i > FM_j$ if $FM_i \in C_u$, $FM_j \in C_v$, and $u < v$. This means that the degree of risk of FM_i is larger than FM_j .

For convenience, the ordinal risk classes of FMs derived from $RO^{(k)}$ are denoted as $\{C_1^k, C_2^k, \dots, C_q^k\}$ and the ordinal risk classes of FMs derived from $RO^{(c)}$ are denoted as $\{C_1^c, C_2^c, \dots, C_q^c\}$.

C. Consensus Measure in the FMs Ordinal Classification-Based FMEA Problem

In general, two different approaches can be adopted in consensus models to determine the consensus level among a group

of individuals: 1) Computing the deviations between the individual and collective orders of the alternatives, and 2) calculating the distances between the individual and collective evaluations or decision matrices [45]. However, in the ordinal classification-based FMEA problem of FMs, FMEA team members are only concerned with the risk classification results of FMs rather than producing a complete risk order of FMs. The extant consensus measure method cannot reflect the essence of the FM ordinal classification-based FMEA problem. Therefore, a novel consensus measure based on distances between individual and collective risk classification results is presented below.

Let $R = (r_1, r_2, \dots, r_n)^T$ be a vector which is used to describe the ordinal risk classes of FMs, where

$$r_i = t \quad (12)$$

if $FM_i \in C_t$ ($i = 1, \dots, n; t = 1, \dots, q$).

Using (12), $R^{(k)} = (r_1^{(k)}, \dots, r_n^{(k)})^T$ and $R^{(c)} = (r_1^{(c)}, \dots, r_n^{(c)})^T$ can be, respectively, generated from $\{C_1^k, C_2^k, \dots, C_q^k\}$ and $\{C_1^c, C_2^c, \dots, C_q^c\}$.

Definition 6: The consensus level of TM_k is defined by

$$CL(V^{(k)}) = \frac{1}{n \cdot (q-1)} \sum_{i=1}^n |r_i^{(k)} - r_i^{(c)}|. \quad (13)$$

The consensus level of $\{TM_1, TM_2, \dots, TM_m\}$ is defined by

$$\begin{aligned} CL\{V^{(1)}, \dots, V^{(m)}\} &= \frac{1}{m} \sum_{k=1}^m CL(V^{(k)}) \\ &= \frac{1}{m \times n \times (q-1)} \sum_{k=1}^m \sum_{i=1}^n |r_i^{(k)} - r_i^{(c)}|. \end{aligned} \quad (14)$$

Clearly, $CL\{V^{(1)}, \dots, V^{(m)}\} \in [0, 1]$. If $CL\{V^{(1)}, \dots, V^{(m)}\} = 0$, then all FMEA team members reach a unanimous consensus regarding the obtained collective ordinal risk classes of FMs. Otherwise, a smaller $CL\{V^{(1)}, \dots, V^{(m)}\}$ value indicates a higher consensus level among FMEA team members.

D. Consensus-Driven Optimization-Based Model

Naturally, it is expected for the consensus level among all FMEA members to be as high as possible. That is

$$\begin{aligned} \min_w CL\{V^{(1)}, \dots, V^{(m)}\} &= \frac{1}{m \times n \times (q-1)} \\ &\times \sum_{k=1}^m \sum_{i=1}^n |r_i^{(k)} - r_i^{(c)}|. \end{aligned} \quad (15)$$

Based on this idea, we present the following optimization model:

$$\begin{aligned} \min_w CL\{V^{(1)}, \dots, V^{(m)}\} \\ = \frac{1}{m \times n \times (q-1)} \sum_{k=1}^m \sum_{i=1}^n |r_i^{(k)} - r_i^{(c)}| \end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{aligned} & v_{ij}^{(c)} = F_{\lambda}(v_{ij}^{(1)}, v_{ij}^{(2)}, \dots, v_{ij}^{(m)}), t = 0, 1, \dots, g \quad (a) \\ & pv_i^{(k)} = \sum_{j=1}^y w_j \cdot \Delta^{-1}(E(v_{ij}^{(k)})), k = 1, 2, \dots, m; \\ & i = 1, 2, \dots, n \quad (b) \\ & pv_i^{(c)} = \sum_{j=1}^y w_j \cdot \Delta^{-1}(E(v_{ij}^{(c)})), i = 1, 2, \dots, n \quad (c) \\ & ro_i^{(k)} = j, \text{ if } pv_i^{(k)} \text{ is the } j\text{th largest value in} \\ & \{pv_1^{(k)}, \dots, pv_n^{(k)}\} \quad (d) \\ & ro_i^{(c)} = j, \text{ if } pv_i^{(c)} \text{ is the } j\text{th largest value in} \\ & \{pv_1^{(c)}, \dots, pv_n^{(c)}\} \quad (e) \\ & r_i^{(k)} = \begin{cases} 1, & ro_i^{(k)} \leq T_1 \\ t, & T_1 + \dots + T_{t-1} + 1 \leq ro_i^{(k)} \\ & \leq T_1 + \dots + T_t \\ q, & ro_i^{(k)} < n - T_q \end{cases} \quad (f) \\ & r_i^{(c)} = \begin{cases} 1, & ro_i^{(c)} \leq T_1 \\ t, & T_1 + \dots + T_{t-1} + 1 \leq ro_i^{(c)} \\ & \leq T_1 + \dots + T_t \\ q, & ro_i^{(c)} < n - T_q \end{cases} \quad (g) \\ & w \in \Omega, \sum_{i=1}^y w_i = 1, w_i \geq 0 \quad (h) \end{aligned} \right. \quad (16)
\end{aligned}$$

In model (16), formula (a) is the aggregation operation that is used to yield $V^{(c)}$. Formulae (b) and (c) are utilized to produce the individual and collective preference vectors of FMs, respectively. Formulae (d) and (e) are utilized to produce the individual and collective risk orders of FMs, respectively. Formulae (f) and (g) are applied to generate the individual and collective ordinal classification vectors of FMs, respectively.

Before solving model (16), Lemma 1 is presented.

Lemma 1: Let $\theta^{(k)} = \{\theta_1^{(k)}, \theta_2^{(k)}, \dots, \theta_{q-1}^{(k)}\}$ be a set of parameters, where $g > \theta_i^{(k)} > \theta_{i+1}^{(k)} \geq 0$. The following condition is satisfied:

$$r_i^{(k)} = \begin{cases} 1, & \text{if } pv_i^{(k)} < \theta_1^{(k)} \\ t, & \text{if } \theta_t^{(k)} < pv_i^{(k)} \leq \theta_{t+1}^{(k)}, 2 \leq t \leq q-1 \\ q, & \text{if } pv_i^{(k)} \leq \theta_{q-1}^{(k)} \end{cases} \quad (17)$$

The consensus level of TM_k can then be computed by the following:

$$CL(V^{(k)}) = \frac{1}{n \cdot (q-1)} \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}| \quad (18)$$

and the consensus level among $\{TM_1, TM_2, \dots, TM_m\}$ can be computed by the following:

$$\begin{aligned}
CL\{V^{(1)}, \dots, V^{(m)}\} &= \frac{1}{m} \sum_{k=1}^m CL(V^{(k)}) \\
&= \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}| \quad (19)
\end{aligned}$$

where $x_{ij}^{(k)}, x_{ij}^{(c)} \in \{0, 1\}$, and determined by

$$\begin{cases} \theta_j^{(k)} - pv_i^{(k)} < x_{ij}^{(k)} \cdot \Re \\ \theta_j^{(k)} - pv_i^{(k)} \geq (x_{ij}^{(k)} - 1) \cdot \Re \end{cases} \quad (20)$$

and

$$\begin{cases} \theta_j^{(c)} - pv_i^{(c)} < x_{ij}^{(c)} \cdot \Re \\ \theta_j^{(c)} - pv_i^{(c)} \geq (x_{ij}^{(c)} - 1) \cdot \Re \end{cases} \quad (21)$$

where \Re is an adequately large number.

Meanwhile, $x_{ij}^{(k)}$ and $x_{ij}^{(c)}$ should satisfy the following conditions:

$$\begin{aligned} & \sum_{i=1}^n x_{ij}^{(k)} = T_{j+1} + T_{j+2} + \dots + T_q, k = 1, 2, \dots, m, \\ & j = 1, 2, \dots, q-1 \end{aligned} \quad (22)$$

$$\sum_{i=1}^n x_{ij}^{(c)} = T_{j+1} + T_{j+2} + \dots + T_q, j = 1, 2, \dots, q-1. \quad (23)$$

Proof: In (20), we have $x_{ij}^{(k)} = 0$ if $pv_i^{(k)} > \theta_j^{(k)}$, and $x_{ij}^{(k)} = 1$ if $pv_i^{(k)} < \theta_j^{(k)}$. In (21), we have $x_{ij}^{(c)} = 0$ if $pv_i^{(c)} > \theta_j^{(c)}$, and $x_{ij}^{(c)} = 1$ if $pv_i^{(c)} < \theta_j^{(c)}$. Thus, we can obtain that $r_i^{(k)} = 1 + \sum_{j=1}^{q-1} x_{ij}^{(k)}$ and $r_i^{(c)} = 1 + \sum_{j=1}^{q-1} x_{ij}^{(c)}$. Thus, $CL(V^{(k)}) = \frac{1}{n \cdot (q-1)} \sum_{i=1}^n |r_i^{(k)} - r_i^{(c)}| = \frac{1}{n \cdot (q-1)} \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}|$ and $CL\{V^{(1)}, \dots, V^{(m)}\} = \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}|$. This completes the proof of Lemma 1.

For simplification, let $M = \{1, 2, \dots, m\}$, $N = \{1, 2, \dots, n\}$, $Q = \{1, 2, \dots, q-1\}$, and $Y = \{1, 2, \dots, y\}$.

Theorem 1: Model (16) can be converted into the following model:

$$\begin{aligned} & \min_w \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}| \\ & \left\{ \begin{aligned} & p_{ij,t}^{(c)} = F_{\lambda}(\hat{p}_{ij,t}^{(1)}, \hat{p}_{ij,t}^{(2)}, \dots, \hat{p}_{ij,t}^{(m)}), t = 0, 1, \dots, g \quad (a) \\ & pv_i^{(k)} = \sum_{j=1}^y w_j \cdot \Delta^{-1}(E(v_{ij}^{(k)})), k \in M, i \in N \quad (b) \\ & pv_i^{(c)} = \sum_{j=1}^y w_j \cdot \Delta^{-1}(E(v_{ij}^{(c)})), k \in M, i \in N \quad (c) \\ & \theta_j^{(k)} - pv_i^{(k)} < x_{ij}^{(k)} \cdot \Re, k \in M, i \in N, j \in Q \quad (d) \\ & \theta_j^{(k)} - pv_i^{(k)} \geq (x_{ij}^{(k)} - 1) \cdot \Re, k \in M, i \in N, j \in Q \quad (e) \\ & \theta_j^{(c)} - pv_i^{(c)} < x_{ij}^{(c)} \cdot \Re, i \in N, j \in Q \quad (f) \\ & \theta_j^{(c)} - pv_i^{(c)} \geq (x_{ij}^{(c)} - 1) \cdot \Re, i \in N, j \in Q \quad (g) \\ & \sum_{i=1}^n x_{ij}^{(k)} = T_{j+1} + T_{j+2} + \dots + T_q, k \in M, j \in Q \quad (h) \\ & \sum_{i=1}^n x_{ij}^{(c)} = T_{j+1} + T_{j+2} + \dots + T_q, j \in Q \quad (i) \\ & w \in \Omega, \sum_{i=1}^y w_i = 1, w_i \geq 0 \quad (j) \\ & x_{ij}^{(k)}, x_{ij}^{(c)} \in \{0, 1\}, k \in M, i \in N, j \in Q \quad (k) \end{aligned} \right. \quad (24)
\end{aligned}$$

where $\hat{p}_{ij,t}^{(k)}$ is obtained from $v_{ij}^{(k)}$ using (6).

Theorem 1 can be directly obtained from Lemma 1. We can thus omit the proof of this theorem.

Model (24) is denoted as M_1 . In model M_1 , $w = (w_1, w_2, \dots, w_y)^T$ are decision variables. Solving model M_1 , we can obtain the optimal solution to $w = (w_1, w_2, \dots, w_y)^T$.

Theorem 2: In model M_1 , we change the objective function into $\min_w \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} b_{ij}^{(k)}$, and add the following constraints: $x_{ij}^{(k)} - x_{ij}^{(c)} \leq b_{ij}^{(k)}$, $-x_{ij}^{(k)} + x_{ij}^{(c)} \leq b_{ij}^{(k)}$, and $b_{ij}^{(k)} \geq 0$, thus formulating a new optimization model denoted as model M_2 . We also note that models M_1 and M_2 are equivalent.

Proof: In model M_2 , $x_{ij}^{(k)} - x_{ij}^{(c)} \leq b_{ij}^{(k)}$ and $-x_{ij}^{(k)} + x_{ij}^{(c)} \leq b_{ij}^{(k)}$ guarantee that $|x_{ij}^{(k)} - x_{ij}^{(c)}| \leq b_{ij}^{(k)}$. The objective function achieves its optimum value only when $|x_{ij}^{(k)} - x_{ij}^{(c)}| = b_{ij}^{(k)}$. Thus, model M_1 can be equivalently converted into model M_2 . This completes the proof of Theorem 2.

Theorem 2 shows that the optimal weights of risk factors in model M_1 can be obtained by solving model M_2 . Model M_2 is a 0–1 mixed linear programming model, which can be easily solved by using diverse mathematical software toolboxes.

VI. CONSENSUS-REACHING PROCESS WITH MINIMUM ADJUSTMENT DISTANCE

In this section, we design a model for supporting consensus reaching processes to help FMEA members achieving a collective and consensual ordinal classification of FMs. First, a consensus rule with a minimum adjustment distance was defined (see Section VI-A). Second, we devise an algorithmic approach to model the consensus-reaching process among FMEA participants (see Section VI-B).

A. Consensus Rule With Minimum Adjustment Distance

Let $\overline{V}^{(k)} = (\overline{v}_{ij}^{(k)})_{n \times y}$ ($k \in M$) be the adjusted HFLAM associated with $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$. Naturally, we hope to minimize the adjustment distance in the consensus-reaching process, i.e.

$$\min \sum_{k=1}^m d(V^{(k)}, \overline{V}^{(k)}) \quad (25)$$

where $d(V^{(k)}, \overline{V}^{(k)})$ signifies the deviation measure between $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ and $\overline{V}^{(k)} = (\overline{v}_{ij}^{(k)})_{n \times y}$, which can be computed as follows:

$$d(V^{(k)}, \overline{V}^{(k)}) = \frac{1}{n \times y} \sum_{i=1}^n \sum_{j=1}^y \frac{|\Delta^{-1}(E(v_{ij}^{(k)})) - \Delta^{-1}(E(\overline{v}_{ij}^{(k)}))|}{g} \quad (26)$$

Meanwhile, the predefined consensus level among all FMEA team members should be guaranteed, i.e.

$$\text{CL}\{\overline{V}^{(1)}, \dots, \overline{V}^{(m)}\} = \frac{1}{m} \sum_{k=1}^m \text{CL}(\overline{V}^{(k)}) \leq \varepsilon \quad (27)$$

where $\varepsilon \in [0, 1]$ is the established consensus threshold.

Following the above basic ideas, we build an optimization-based consensus model:

$$\begin{aligned} \min & \sum_{k=1}^m d(V^{(k)}, \overline{V}^{(k)}) \\ \text{s.t.} & \begin{cases} \overline{v}_{ij}^{(c)} = F_{\pi}(\overline{v}_{ij}^{(1)}, \dots, \overline{v}_{ij}^{(m)}), & i \in N, j \in Y \\ \frac{1}{m} \sum_{k=1}^m \text{CL}(\overline{V}^{(k)}) \leq \varepsilon \\ \overline{v}_{ij}^{(k)} \in M^S, & k \in M; i \in N; j \in Y. \end{cases} \end{aligned} \quad (28)$$

Model (28) is denoted as M_3 . Different optimization-based consensus models will be generated when setting different aggregation operators/functions (F_{π}) in M_3 . The use of different aggregation functions does not alter the essence of the proposed consensus-based FMEA framework. Particularly, in linguistic GDM problems, the HFLWA is a widely used aggregation function. Without loss of generality, this paper investigates the optimization-based consensus model based on HFLWA in detail.

When HFLWA is selecting as the aggregation function of model M_3 , it can be further instantiated as the following model:

$$\begin{aligned} \min & \frac{1}{n \times y \times g} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^y |\Delta^{-1}(E(v_{ij}^{(k)})) - a_{ij}^{(k)}| \\ \text{s.t.} & \begin{cases} a_{ij}^{(c)} = \sum_{k=1}^m \lambda_k \cdot a_{ij}^{(k)}, & i \in N, j \in Y & (a) \\ \text{pv}_i^{(k)} = \sum_{j=1}^y w_j \cdot a_{ij}^{(k)}, & k \in M, i \in N, j \in Y & (b) \\ \text{pv}_i^{(c)} = \sum_{j=1}^y w_j \cdot a_{ij}^{(c)}, & i \in N & (c) \\ \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}| \leq \varepsilon & (d) \\ \theta_j^{(k)} - \text{pv}_i^{(k)} < x_{ij}^{(k)} \cdot \Re, & k \in M, i \in N, j \in Q & (e) \\ \theta_j^{(k)} - \text{pv}_i^{(k)} \geq (x_{ij}^{(k)} - 1) \cdot \Re, & k \in M, i \in N, j \in Q & (f) \\ \theta_j^{(c)} - \text{pv}_i^{(c)} < x_{ij}^{(c)} \cdot \Re, & i \in N, j \in Q & (g) \\ \theta_j^{(c)} - \text{pv}_i^{(c)} \geq (x_{ij}^{(c)} - 1) \cdot \Re, & i \in N, j \in Q & (h) \\ \sum_{i=1}^m x_{ij}^{(k)} = T_{j+1} + T_{j+2} + \dots + T_q, & k \in M, j \in Q & (i) \\ \sum_{i=1}^m x_{ij}^{(c)} = T_{j+1} + T_{j+2} + \dots + T_q, & k \in M, j \in Q & (j) \\ g \geq \theta_1^{(k)} \geq \theta_2^{(k)} \geq \dots \geq \theta_{q-1}^{(k)} \geq 0 & (k) \\ g \geq \theta_1^{(c)} \geq \theta_2^{(c)} \geq \dots \geq \theta_{q-1}^{(c)} \geq 0 & (l) \\ 0 \leq a_{ij}^{(k)} \leq g, & k \in M, i \in N, j \in Y & (m) \\ x_{ij}^{(k)}, x_{ij}^{(c)} \in \{0, 1\}, & k \in M, i \in N, j \in Q & (n) \end{cases} \end{aligned} \quad (29)$$

where $\Delta(a_{ij}^{(k)}) = E(v_{ij}^{(k)})$ and $\Delta(a_{ij}^{(c)}) = E(v_{ij}^{(c)})$.

Model (29) is denoted as M_4 .

Theorem 4: Model M_4 can be converted into a mixed 0–1 linear programming model:

$$\begin{aligned}
 \min & \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^y b_{ij}^{(k)} \\
 \text{s.t.} & \begin{cases}
 \Delta^{-1}(E(v_{ij}^{(k)})) - a_{ij}^{(k)} \leq b_{ij}^{(k)}, k \in M, i \in N, j \in Y & (a) \\
 -\Delta^{-1}(E(v_{ij}^{(k)})) + a_{ij}^{(k)} \leq b_{ij}^{(k)}, k \in M, i \in N, j \in Y & (b) \\
 a_{ij}^{(c)} = \sum_{k=1}^m \lambda_k \cdot a_{ij}^{(k)}, i \in N, j \in Y & (c) \\
 \text{pv}_i^{(k)} = \sum_{j=1}^y w_j \cdot a_{ij}^{(k)}, k \in M, i \in N, j \in Y & (d) \\
 \text{pv}_i^{(c)} = \sum_{j=1}^y w_j \cdot a_{ij}^{(c)}, i \in N & (e) \\
 \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} f_{ij}^{(k)} \leq \varepsilon & (f) \\
 x_{ij}^{(k)} - x_{ij}^{(c)} \leq f_{ij}^{(k)}, k \in M, i \in N, j \in Q & (g) \\
 -x_{ij}^{(k)} + x_{ij}^{(c)} \leq f_{ij}^{(k)}, k \in M, i \in N, j \in Q & (h) \\
 \theta_j^{(k)} - \text{pv}_i^{(k)} < x_{ij}^{(k)} \cdot \mathfrak{R}, k \in M, i \in N, j \in Q & (i) \\
 \theta_j^{(k)} - \text{pv}_i^{(k)} \geq (x_{ij}^{(k)} - 1) \cdot \mathfrak{R}, k \in M, i \in N, j \in Q & (j) \\
 \theta_j^{(c)} - \text{pv}_i^{(c)} < x_{ij}^{(c)} \cdot \mathfrak{R}, i \in N, j \in Q & (k) \\
 \theta_j^{(c)} - \text{pv}_i^{(c)} \geq (x_{ij}^{(c)} - 1) \cdot \mathfrak{R}, i \in N, j \in Q & (l) \\
 \sum_{i=1}^m x_{ij}^{(k)} = T_{j+1} + T_{j+2} + \dots + T_q, k \in M, j \in Q & (m) \\
 \sum_{i=1}^m x_{ij}^{(c)} = T_{j+1} + T_{j+2} + \dots + T_q, k \in M, j \in Q & (n) \\
 g \geq \theta_1^{(k)} \geq \theta_2^{(k)} \geq \dots \geq \theta_{q-1}^{(k)} \geq 0 & (o) \\
 g \geq \theta_1^{(c)} \geq \theta_2^{(c)} \geq \dots \geq \theta_{q-1}^{(c)} \geq 0 & (p) \\
 0 \leq a_{ij}^{(k)} \leq g, k \in M, i \in N, j \in Y & (q) \\
 x_{ij}^{(k)}, x_{ij}^{(c)} \in \{0, 1\}, k \in M, i \in N, j \in Q. & (r)
 \end{cases} \\
 & (30)
 \end{aligned}$$

Proof: In model M_4 , constraints (a) and (b) guarantee that $|\Delta^{-1}(E(v_{ij}^{(k)})) - a_{ij}^{(k)}| \leq b_{ij}^{(k)}$. The objective function achieves optimum value only when $|\Delta^{-1}(E(v_{ij}^{(k)})) - a_{ij}^{(k)}| = b_{ij}^{(k)}$. Moreover, constraints (g) and (h) guarantee that $|x_{ij}^{(k)} - x_{ij}^{(c)}| \leq f_{ij}^{(k)}$. According to (f), we have that $\frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} |x_{ij}^{(k)} - x_{ij}^{(c)}| \leq \frac{1}{m \cdot n \cdot (q-1)} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^{q-1} f_{ij}^{(k)} \leq \varepsilon$. Thus, model M_4 can be converted into model (30).

Model (30) is denoted as M_5 . Theorem 4 implies that the optimum solution of model M_4 can be generated by solving model M_5 .

B. Consensus-Reaching Algorithm

Solution of the model M_5 , we yield the optimal solution to $A^{(k)} = (a_{ij}^{(k)})_{n \times y}$, which is denoted as $A^{(k,*)} = (a_{ij}^{(k,*)})_{n \times y}$. Furthermore, we can obtain that $E(v_{ij}^{(k,*)}) = \Delta(a_{ij}^{(k,*)})$. Then,

TABLE I
CONSENSUS-REACHING ALGORITHM

Input: The PHFLAMs $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ ($k = 1, 2, \dots, m$), the weights of FMEA members $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$, the incomplete information regarding the weights of risk factors Ω , T_i ($i = 1, 2, \dots, q$), and the consensus threshold ε .

Output: Adjusted PHFLAMs $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times y}$ ($k = 1, 2, \dots, m$), and the ordinal risk classes of FMs, C_1, C_2, \dots, C_q .

Step 1: Let $z = 0$, $V^{(k,z)} = V^{(k)}$.

Step 2: According to Eq. (8), $V^{(k,z)} = (v_{ij}^{(k,z)})_{n \times y}$ ($k = 1, 2, \dots, m$) are aggregated into a collective PHFLAM $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times y}$.

Step 3: Using M_2 to generate the optimal weights of risk factors, which is denoted as $w^z = (w_{1,z}, w_{2,z}, \dots, w_{y,z})^T$. Meanwhile, the optimal value objective function can be obtained as opv_z .

Step 4: If $opv_z \leq \varepsilon$, then go to **Step 7**, otherwise, continue with the next step.

Step 5: Applying the optimization-based consensus model M_5 to generate $A^{(k,z,*)} = (a_{ij}^{(k,z,*)})_{n \times y}$. We can then obtain that $E(v_{ij}^{(k,z,*)}) = \Delta(a_{ij}^{(k,z,*)})$. When constructing the adjusted HFLAMs $V^{(k,z+1)} = (v_{ij}^{(k,z+1)})_{n \times y}$ ($k = 1, 2, \dots, m$), the following direction rules are applied to guide participants:

(a) If $E(v_{ij}^{(k,z)}) < E(v_{ij}^{(k,z,*)})$, then TM_k should increase their assessments regarding FM_i with respect to RF_j .

(b) If $E(v_{ij}^{(k,z)}) > E(v_{ij}^{(k,z,*)})$, then TM_k should decrease their assessments regarding FM_i with respect to RF_j .

(c) If $E(v_{ij}^{(k,z)}) = E(v_{ij}^{(k,z,*)})$, then TM_k should maintain unchanged their assessments regarding FM_i with respect to RF_j .

Let $z = z + 1$.

Step 6: Using Eq. (8), $V^{(k,z)} = (v_{ij}^{(k,z)})_{n \times y}$ can be aggregated into a collective PHFLAM $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times y}$. Then, according to Eq. (11), we can obtain that $\{C_{1,z}^k, C_{2,z}^k, \dots, C_{q,z}^k\}$ and $\{C_{1,z}^c, C_{2,z}^c, \dots, C_{q,z}^c\}$ from $V^{(k,z)} = (v_{ij}^{(k,z)})_{n \times y}$ and $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times y}$, respectively. Using Eq. (14), the consensus level among FMEA members CL_z is obtained. If $CL_z \leq \varepsilon$, go to **Step 7**, otherwise, go to **Step 2**.

Step 7: Let $\overline{V^{(k)}} = V^{(k,z)}$ ($k = 1, 2, \dots, m$), and $C_i = C_{i,z}^c$ ($i = 1, 2, \dots, q$). Output $\overline{V^{(k)}}$ and C_1, C_2, \dots, C_q .

$E(v_{ij}^{(k,*)})$ are used as reference information for guiding FMEA team members in modifying their preferences. When constructing $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times y}$, we advise that

- R.1. If $E(v_{ij}^{(k)}) < E(v_{ij}^{(k,*)})$, we advise that TM_k increase their assessments regarding FM_i with respect to RF_j .
- R.2. If $E(v_{ij}^{(k)}) > E(v_{ij}^{(k,*)})$, we advise that TM_k decrease their assessments regarding FM_i with respect to RF_j .
- R.3. If $E(v_{ij}^{(k)}) = E(v_{ij}^{(k,*)})$, then TM_k should maintain their assessments unchanged regarding FM_i with respect to RF_j .

The details of the consensus-reaching process are described as follows: After the FMEA members provide individual PHFLAMs, $V^{(k)} = (v_{ij}^{(k)})_{n \times y}$ ($k \in M$), and then these PHFLAMs

TABLE II
FMEA OF THE TREATMENT PLANNING STAGE IN PROTON BEAM RADIOTHERAPY

Items	Failure modes	Failure causes	Failure effects
FM_1	Outdated representation of the anatomy (single CT scan)	Anatomical changes (related to time delay)	Wrong dose distribution/ wrong dose delivery
FM_2	Inaccurate delineation	Human error	Wrong dose distribution
FM_3	Incorrect HU number manual assignment	Human error or lack of documentation from the referring clinicians	Wrong dose distribution
FM_4	Wrong definition of couch origin of coordinates (small amount, in terms of 2-3 mm)	Human error	Unintended normal tissue irradiated and CTV missing
FM_5	Improper selection of gantry angle/couch rotation: beam stopping against OARs	Inadequate operator skill	Low plan robustness (range uncertainty)
FM_6	Improper selection of gantry angle/ couch rotation: beam passing through unstable tissues	Inadequate operator skill	Low plan robustness (range uncertainty)

are aggregated into a collective PHFLAM, $V^{(c)} = (v_{ij}^{(c)})_{n \times y}$. Following this, model M_2 is applied to generate the weights of risk factors, $w = (w_1, w_2, \dots, w_y)^T$, from the incomplete weight information (as described in Section V). Meanwhile, the consensus level, CL, the individual and collective ordinal risk classes of FMs $\{C_1^k, C_2^k, \dots, C_q^k\}$ and $\{C_1^c, C_2^c, \dots, C_q^c\}$ can be also obtained. If the current consensus level CL is acceptable, the consensus-reaching process terminates. Otherwise, model M_4 is adopted to generate $A^{(k,*)} = (a_{ij}^{(k,*)})_{n \times y}$. Furthermore, it is proven that $E(v_{ij}^{(k,*)}) = \Delta(a_{ij}^{(k,*)})$. Subsequently, $E(v_{ij}^{(k,*)})$ results from the application of M_5 , and are used for guiding participants in revising and providing updated PHFLAMs $\overline{V}^{(k)} = (v_{ij}^{(k)})_{n \times y}$ using R.1, R.2, and R.3. This procedure is followed until the predefined consensus level among FMEA members is achieved.

Herein, we design a consensus-reaching algorithm to describe the consensus-reaching process in Table I.

VII. CASE STUDY

This section shows the practical use of the proposed consensus-based FMEA approach to the problem of treatment planning in scanned proton beam radiotherapy (SPBR), which is adopted from Cantone *et al.* [7]. Their work was focused on how to generate a complete ranking of FMs from the most to the least risky (i.e., least to most reliable) FM. Moreover, the consensus issue among FMEA experts is not addressed in Cantone *et al.*'s approach. This study implements some revisions regarding this example to better show the use of the proposed consensus-based FMEA approach.

Active scanned proton beam (SPB) has been extensively used in radiation therapy, which adopts the physical interaction properties of the particles with human tissue and an advanced delivery modality to improve treatment results. However, accidental exposures are increasingly frequent nowadays in the SPBR implementation process owing to the increased complexity related to the technological and various uncontrollable factors. To effectively classify and deal with the potential risks of accidental exposures at diverse levels when using actively SPB, the proposed consensus-based MAGDM approach is utilized to identify

the critical potential FMs that might occur during a radiotherapy treatment.

In Cantone *et al.* [7], 44 FMs were initially identified during the SPBR process. For ease of illustration, six FMs (FM_1, FM_2, \dots, FM_6) with high RPN values are chosen in this paper for further detailed illustration. The six FMs and their causes and effects are listed in Table II. A multidisciplinary FMEA team with three experts is formed to classify the six FMs into three ordinal risk classes, ranked from the highest to the lowest levels of risk (i.e., high, medium, and low) involved in the proton beam radiation therapy, with each one consisting of two FMs. The three FMEA experts are denoted as TM_1, TM_2 , and TM_3 , respectively. Considering their domain experiences and knowledge, the weights of the three FMEA experts are set as 0.35, 0.4, and 0.25. The risk factors used to evaluate the six FMs are O (occurrence), S (severity), and D (detection). It should be noted that the proposed consensus-based FMEA model is capable of dealing with as many FMs and risk factors as the FMEA experts wish to consider in the risk analysis process.

In practice, consensual decisions are crucial for implementing a highly accepted group solution to the FMEA problem. In what follows, the proposed consensus-based FMEA model is utilized to solve the healthcare risk assessment problem.

First, the assessment information of the FMs with respect to the three risk factors is modeled using a nine-grade linguistic term set S , which is provided as follows:

$$\begin{aligned}
 S = \{ & s_0 = \text{Absolutely Low (AL)}, s_1 = \text{Very Low (VL)}, \\
 & s_2 = \text{Low (L)}, \\
 & s_3 = \text{Moderately Low (ML)}, s_4 = \text{Moderate (M)}, \\
 & s_5 = \text{Moderately High (MH)}, \\
 & s_6 = \text{High (H)}, \\
 & s_7 = \text{Very High (VH)}, s_8 = \text{Absolutely High (AH)} \}.
 \end{aligned}$$

In this case study, we set the consensus threshold to $\varepsilon = 0.15$. The three individual PHFLAMs on the six FMs against every risk factor are obtained as presented in Tables III–V.

TABLE III
PHFLAM $V^{(1)}$ PROVIDED BY TM_1

	O	S	D
FM_1	$\{(s_6, 0.4), (s_7, 0.6)\}$	$\{(s_5, 0.35), (s_6, 0.4), (s_7, 0.25)\}$	$\{(0.4, s_6), (s_7, 0.3), (s_8, 0.3)\}$
FM_2	$\{(s_7, 0.55), (s_8, 0.45)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_5, 0.45), (s_6, 0.3), (s_7, 0.25)\}$
FM_3	$\{(s_6, 0.35), (s_7, 0.65)\}$	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$
FM_4	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_5, 0.42), (s_6, 0.58)\}$	$\{(s_5, 0.52), (s_6, 0.3), (s_7, 0.18)\}$
FM_5	$\{(s_4, 0.35), (s_5, 0.65)\}$	$\{(s_4, 0.3), (s_5, 0.45), (s_6, 0.25)\}$	$\{(s_5, 0.65), (s_6, 0.35)\}$
FM_6	$\{(s_7, 0.55), (s_8, 0.45)\}$	$\{(s_4, 0.3), (s_5, 0.35), (s_6, 0.35)\}$	$\{(s_4, 0.3), (s_5, 0.7)\}$

TABLE IV
PHFLAM $V^{(2)}$ PROVIDED BY TM_2

	O	S	D
FM_1	$\{(s_4, 0.42), (s_5, 0.58)\}$	$\{(s_5, 0.42), (s_6, 0.58)\}$	$\{(s_4, 0.62), (s_5, 0.38)\}$
FM_2	$\{(s_5, 0.3), (s_6, 0.45), (s_7, 0.25)\}$	$\{(s_4, 0.32), (s_5, 0.68)\}$	$\{(s_4, 0.25), (s_5, 0.5), (s_6, 0.25)\}$
FM_3	$\{(s_6, 0.65), (s_7, 0.35)\}$	$\{(s_5, 0.35), (s_6, 0.65)\}$	$\{(s_5, 0.4), (s_6, 0.1), (s_7, 0.5)\}$
FM_4	$\{(s_5, 0.1), (s_6, 0.2), (s_7, 0.7)\}$	$\{(s_6, 0.2), (s_7, 0.3), (s_8, 0.5)\}$	$\{(s_6, 0.7), (s_7, 0.3)\}$
FM_5	$\{(s_6, 0.43), (s_7, 0.57)\}$	$\{(s_5, 0.2), (s_6, 0.8)\}$	$\{(s_4, 0.5), (s_5, 0.3), (s_6, 0.2)\}$
FM_6	$\{(s_5, 0.1), (s_6, 0.9)\}$	$\{(s_4, 0.2), (s_5, 0.5), (s_6, 0.3)\}$	$\{(s_5, 0.6), (s_6, 0.4)\}$

TABLE V
PHFLAM $V^{(3)}$ PROVIDED BY TM_3

	O	S	D
FM_1	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_5, 0.3), (s_6, 0.4), (s_7, 0.3)\}$	$\{(s_4, 0.65), (s_5, 0.35)\}$
FM_2	$\{(s_5, 0.2), (s_6, 0.25), (s_7, 0.35), (s_8, 0.2)\}$	$\{(s_5, 0.5), (s_6, 0.2), (s_7, 0.3)\}$	$\{(s_5, 0.4), (s_6, 0.2), (s_7, 0.25), (s_8, 0.15)\}$
FM_3	$\{(s_5, 0.2), (s_6, 0.45), (s_7, 0.35)\}$	$\{(s_4, 0.6), (s_5, 0.4)\}$	$\{(s_5, 0.48), (s_6, 0.52)\}$
FM_4	$\{(s_5, 0.38), (s_6, 0.62)\}$	$\{(s_4, 0.35), (s_5, 0.2), (s_6, 0.45)\}$	$\{(s_4, 0.25), (s_5, 0.75)\}$
FM_5	$\{(s_5, 0.2), (s_6, 0.3), (s_7, 0.5)\}$	$\{(s_5, 0.3), (s_6, 0.4), (s_7, 0.3)\}$	$\{(s_5, 0.7), (s_6, 0.3)\}$
FM_6	$\{(s_6, 0.4), (s_7, 0.25), (s_8, 0.35)\}$	$\{(s_5, 0.65), (s_6, 0.35)\}$	$\{(s_6, 0.3), (s_7, 0.45), (s_8, 0.25)\}$

TABLE VI
OPTIMALLY ADJUSTED TWO-TUPLE LINGUISTIC ASSESSMENT MATRIX $F^{(1,*)}$

	O	S	D
FM_1	$(s_7, -0.4)$	$(s_3, 0.0575)$	$(s_7, -0.1)$
FM_2	$(s_7, 0.45)$	$(s_6, -0.4)$	$(s_6, -0.2)$
FM_3	$(s_7, -0.35)$	$(s_4, 0.2887)$	$(s_6, -0.4)$
FM_4	$(s_4, 0.3)$	$(s_6, -0.42)$	$(s_6, -0.34)$
FM_5	$(s_5, -0.35)$	$(s_6, -0.2425)$	$(s_5, 0.35)$
FM_6	$(s_5, 0.45)$	$(s_5, 0.05)$	$(s_5, -0.3)$

1) By incorporating PHFLAMs $V^{(k)}$ ($k = 1, 2, 3$) into model M_2 , we can obtain the weights of the risk factors, that is $w = (0.25, 0.4, 0.35)^T$.

Meanwhile, the optimal value of the objective function of M_2 is generated, that is, $opv = 0.3889$. This indicates that the predefined consensus level cannot be achieved by optimizing the weights of risk factors owing to $opv < \varepsilon$.

Herein, we consider that $w = (0.25, 0.4, 0.35)^T$ and $V^{(1)}$, $V^{(2)}$, and $V^{(3)}$, as the inputs of the optimization model M_5 , based on which can obtain that $A^{(k,*)} = (a_{ij}^{(k,*)})_{n \times n}$. Furthermore, $A^{(k,*)} = (a_{ij}^{(k,*)})_{n \times n}$ ($k = 1, 2, 3$) are transformed into two-tuple linguistic assessment matrices $F^{(k,*)} = (f_{ij}^{(k,*)})_{n \times n}$, ($k = 1, 2, 3$), where $f_{ij}^{(k,z,*)} = \Delta(a_{ij}^{(k,z,*)})$, which are listed in Tables VI–VIII.

When providing the updated PHFLAMs, $V^{(1,1)} = (v_{ij}^{(1,1)})_{6 \times 3}$, $V^{(2,1)} = (v_{ij}^{(2,1)})_{6 \times 3}$, and $V^{(3,1)} = (v_{ij}^{(3,1)})_{6 \times 3}$, we suggest that

TABLE VII
OPTIMALLY ADJUSTED TWO-TUPLE LINGUISTIC ASSESSMENT MATRIX $F^{(2,*)}$

	O	S	D
FM_1	$(s_5, -0.42)$	$(s_6, -0.42)$	$(s_4, 0.38)$
FM_2	$(s_6, -0.05)$	$(s_5, 0.1175)$	$(s_5, 0)$
FM_3	$(s_6, 0.35)$	$(s_6, -0.35)$	$(s_6, 0.1)$
FM_4	$(s_7, -0.4)$	$(s_7, 0.3)$	$(s_6, 0.3)$
FM_5	$(s_7, -0.43)$	$(s_7, -0.2625)$	$(s_5, -0.3)$
FM_6	$(s_6, -0.1)$	$(s_5, 0.1)$	$(s_3, 0.4)$

TABLE VIII
OPTIMALLY ADJUSTED TWO-TUPLE LINGUISTIC ASSESSMENT MATRIX $F^{(3,*)}$

	O	S	D
FM_1	$(s_6, -0.4)$	$(s_5, 0.4625)$	$(s_4, 0.35)$
FM_2	$(s_6, -0.445)$	$(s_7, -0.0781)$	$(s_6, 0.15)$
FM_3	$(s_5, 0.15)$	$(s_4, 0.4)$	$(s_6, -0.48)$
FM_4	$(s_6, -0.38)$	$(s_5, 0.1)$	$(s_5, -0.25)$
FM_5	$(s_6, 0.3)$	$(s_7, 0)$	$(s_7, 0.3)$
FM_6	$(s_7, -0.05)$	$(s_5, 0.35)$	$(s_7, -0.05)$

TABLE IX
HFLAM $V^{(1,1)}$

	O	S	D
FM_1	$\{(s_6, 0.4), (s_7, 0.6)\}$	$\{(s_3, 0.9425), (s_4, 0.0575)\}$	$\{(0.4, s_6), (s_7, 0.3), (s_8, 0.3)\}$
FM_2	$\{(s_7, 0.55), (s_8, 0.45)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_5, 0.45), (s_6, 0.3), (s_7, 0.25)\}$
FM_3	$\{(s_6, 0.35), (s_7, 0.65)\}$	$\{(s_4, 0.71), (s_5, 0.29)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$
FM_4	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_5, 0.42), (s_6, 0.58)\}$	$\{(s_5, 0.52), (s_6, 0.3), (s_7, 0.18)\}$
FM_5	$\{(s_4, 0.35), (s_5, 0.65)\}$	$\{(s_5, 0.7), (s_6, 0.3)\}$	$\{(s_5, 0.65), (s_6, 0.35)\}$
FM_6	$\{(s_5, 0.55), (s_6, 0.45)\}$	$\{(s_4, 0.3), (s_5, 0.35), (s_6, 0.35)\}$	$\{(s_4, 0.3), (s_5, 0.7)\}$

TABLE X
HFLAM $V^{(2,1)}$

	O	S	D
FM_1	$\{(s_4, 0.42), (s_5, 0.58)\}$	$\{(s_5, 0.42), (s_6, 0.58)\}$	$\{(s_4, 0.62), (s_5, 0.38)\}$
FM_2	$\{(s_5, 0.3), (s_6, 0.45), (s_7, 0.25)\}$	$\{(s_4, 0.1), (s_5, 0.8), (s_6, 0.1)\}$	$\{(s_4, 0.25), (s_5, 0.5), (s_6, 0.25)\}$
FM_3	$\{(s_6, 0.65), (s_7, 0.35)\}$	$\{(s_5, 0.35), (s_6, 0.65)\}$	$\{(s_5, 0.4), (s_6, 0.1), (s_7, 0.5)\}$
FM_4	$\{(s_5, 0.1), (s_6, 0.2), (s_7, 0.7)\}$	$\{(s_6, 0.2), (s_7, 0.3), (s_8, 0.5)\}$	$\{(s_6, 0.7), (s_7, 0.3)\}$
FM_5	$\{(s_6, 0.43), (s_7, 0.57)\}$	$\{(s_5, 0.2), (s_6, 0.8)\}$	$\{(s_4, 0.5), (s_5, 0.3), (s_6, 0.2)\}$
FM_6	$\{(s_5, 0.1), (s_6, 0.9)\}$	$\{(s_4, 0.2), (s_5, 0.5), (s_6, 0.3)\}$	$\{(s_5, 0.6), (s_6, 0.4)\}$

- TM_1 should decrease the assessment values regarding the FM_1 and FM_3 with respect to S owing to $E(v_{12}^{(1)}) > f_{12}^{(1,*)}$ and $E(v_{32}^{(1)}) > f_{32}^{(1,*)}$, and TM_1 should increase the assessment value regarding FM_5 with respect to S owing to the fact that $E(v_{52}^{(1)}) < f_{52}^{(1,*)}$;
- TM_2 should increase the assessment values regarding FM_2 and FM_5 with respect to S owing to the facts that $E(v_{22}^{(2)}) < f_{22}^{(2,*)}$ and $E(v_{52}^{(2)}) < f_{52}^{(2,*)}$;
- TM_3 should decrease the assessment value regarding FM_1 with respect to S owing to $E(v_{12}^{(3)}) < f_{12}^{(3,*)}$, and TM_3 should decrease the assessment value regarding the FM_2 with respect to S owing to the fact that $E(v_{22}^{(3)}) > f_{22}^{(3,*)}$.

Without loss of generality, FMEA members provide their updated HFLAMs, as listed in Tables IX–XI.

Again, according to (14), the consensus level among all FMEA members can be obtained, which is $cl_1 = 0.3333$. This indicates that the predefined consensus level has not been achieved owing to $cl_1 > \varepsilon$.

TABLE XI
HFLAM $V^{(3,1)}$

	O	S	D
FM_1	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_5, 0.53), (s_6, 0.47)\}$	$\{(s_6, 0.65), (s_7, 0.35)\}$
FM_2	$\{(s_1, 0.2), (s_2, 0.25), (s_4, 0.35), (s_7, 0.2)\}$	$\{(s_6, 0.7), (s_7, 0.1), (s_8, 0.2)\}$	$\{(s_1, 0.4), (s_2, 0.2), (s_7, 0.25), (s_8, 0.15)\}$
FM_3	$\{(s_4, 0.2), (s_5, 0.45), (s_6, 0.35)\}$	$\{(s_1, 0.6), (s_2, 0.4)\}$	$\{(s_1, 0.48), (s_6, 0.52)\}$
FM_4	$\{(s_5, 0.38), (s_6, 0.62)\}$	$\{(s_4, 0.35), (s_7, 0.2), (s_8, 0.45)\}$	$\{(s_4, 0.25), (s_7, 0.75)\}$
FM_5	$\{(s_1, 0.2), (s_2, 0.3), (s_7, 0.5)\}$	$\{(s_6, 0.3), (s_7, 0.4), (s_8, 0.3)\}$	$\{(s_1, 0.7), (s_4, 0.3)\}$
FM_6	$\{(s_6, 0.4), (s_7, 0.25), (s_8, 0.35)\}$	$\{(s_5, 0.65), (s_6, 0.35)\}$	$\{(s_6, 0.3), (s_7, 0.45), (s_8, 0.25)\}$

TABLE XII
OPTIMALLY ADJUSTED TWO-TUPLE LINGUISTIC
ASSESSMENT MATRIX $F^{(2,1,*)}$

	O	S	D
FM_1	$(s_5, -0.42)$	$(s_6, -0.42)$	$(s_4, 0.38)$
FM_2	$(s_6, -0.05)$	$(s_5, 0)$	$(s_5, 0)$
FM_3	$(s_6, 0.35)$	$(s_6, -0.35)$	$(s_6, 0.1)$
FM_4	$(s_7, -0.4)$	$(s_7, -0.2259)$	$(s_6, 0.3)$
FM_5	$(s_7, -0.43)$	$(s_7, -0.28)$	$(s_5, -0.3)$
FM_6	$(s_6, -0.1)$	$(s_5, 0.1)$	$(s_3, 0.4)$

TABLE XIII
OPTIMALLY ADJUSTED TWO-TUPLE LINGUISTIC
ASSESSMENT MATRIX $F^{(3,1,*)}$

	O	S	D
FM_1	$(s_6, -0.4)$	$(s_5, 0.31)$	$(s_4, 0.35)$
FM_2	$(s_6, -0.445)$	$(s_7, -0.5)$	$(s_6, 0.15)$
FM_3	$(s_5, 0.15)$	$(s_4, 0.4)$	$(s_6, -0.48)$
FM_4	$(s_6, -0.38)$	$(s_5, 0.1)$	$(s_5, -0.25)$
FM_5	$(s_6, 0.3)$	$(s_7, 0)$	$(s_7, 0.3)$
FM_6	$(s_7, -0.05)$	$(s_5, 0.35)$	$(s_7, -0.05)$

- 1) By incorporating PHFLAMs $V^{(k,1)}$ ($k = 1, 2, 3$) into model M_2 , we can obtain the weights of the risk factors, that is, $w^1 = (0.25, 0.5, 0.25)^T$.

Meanwhile, the optimal value of the objective function of M_2 is generated, that is, $opv_2 = 0.2222$. This indicates that the predefined consensus level cannot be achieved by optimizing the weights of risk factors owing to $opv > \varepsilon$.

Herein, we consider that $w^2 = (0.25, 0.5, 0.25)^T$, and that $V^{(1,1)}$, $V^{(2,1)}$, and $V^{(3,1)}$, as the input of the optimization model M_5 , based on which we can obtain that $A^{(k,1,*)} = (a_{ij}^{(k,1,*)})_{n \times n}$. Furthermore, $A^{(k,1,*)} = (a_{ij}^{(k,1,*)})_{n \times n}$ is transformed into two-tuple linguistic assessment matrices $F^{(k,1,*)} = (f_{ij}^{(k,1,*)})_{n \times n}$ (i.e., $f_{ij}^{(k,1,*)} = \Delta(a_{ij}^{(k,1,*)})$), which are listed in Tables XII and XIII. Correspondingly, $F^{(1,*)} = F^{(1)}$.

When providing the updated HFLAMs $V^{(k,2)}$ ($k = 1, 2, 3$), we suggest that

- the TM_1 of FMEA members maintain their HFLAMs unchanged
- the TM_2 of FMEA members should decrease the assessment value of FM_4 with respect to the risk factor S owing to the fact that $E(v_{42}^{(2,1)}) > f_{42}^{(2,1,*)}$;
- FMEA member TM_3 should decrease the assessment value of FM_1 with respect to risk factor S owing to $E(v_{12}^{(3,1)}) > f_{12}^{(3,1,*)}$.

Without loss of generality, the TM_1 of FMEA members set their HFLAM to $V^{(1,2)} = V^{(1,1)}$. The HFLAM $V^{(2,2)}$ and $V^{(3,2)}$ values respectively provided by TM_2 and TM_3 are listed in Tables XIV and XV, respectively.

TABLE XIV
HFLAM $V^{(2,2)}$

	O	S	D
FM_1	$\{(s_4, 0.42), (s_5, 0.58)\}$	$\{(s_5, 0.42), (s_6, 0.58)\}$	$\{(s_4, 0.62), (s_5, 0.38)\}$
FM_2	$\{(s_5, 0.3), (s_6, 0.45), (s_7, 0.25)\}$	$\{(s_4, 0.1), (s_5, 0.8), (s_6, 0.1)\}$	$\{(s_4, 0.25), (s_5, 0.5), (s_6, 0.25)\}$
FM_3	$\{(s_6, 0.65), (s_7, 0.35)\}$	$\{(s_5, 0.35), (s_6, 0.65)\}$	$\{(s_5, 0.4), (s_6, 0.1), (s_7, 0.5)\}$
FM_4	$\{(s_5, 0.1), (s_6, 0.2), (s_7, 0.7)\}$	$\{(s_6, 0.3), (s_7, 0.7)\}$	$\{(s_6, 0.7), (s_7, 0.3)\}$
FM_5	$\{(s_6, 0.43), (s_7, 0.57)\}$	$\{(s_5, 0.2), (s_6, 0.8)\}$	$\{(s_4, 0.5), (s_5, 0.3), (s_6, 0.2)\}$
FM_6	$\{(s_5, 0.1), (s_6, 0.9)\}$	$\{(s_4, 0.2), (s_5, 0.5), (s_6, 0.3)\}$	$\{(s_3, 0.6), (s_4, 0.4)\}$

TABLE XV
HFLAM $V^{(3,2)}$

	O	S	D
FM_1	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_5, 0.7), (s_6, 0.3)\}$	$\{(s_4, 0.65), (s_5, 0.35)\}$
FM_2	$\{(s_1, 0.2), (s_2, 0.25), (s_4, 0.35), (s_7, 0.2)\}$	$\{(s_6, 0.7), (s_7, 0.1), (s_8, 0.2)\}$	$\{(s_1, 0.4), (s_2, 0.2), (s_7, 0.25), (s_8, 0.15)\}$
FM_3	$\{(s_4, 0.2), (s_5, 0.45), (s_6, 0.35)\}$	$\{(s_1, 0.6), (s_2, 0.4)\}$	$\{(s_1, 0.48), (s_6, 0.52)\}$
FM_4	$\{(s_5, 0.38), (s_6, 0.62)\}$	$\{(s_4, 0.35), (s_7, 0.2), (s_8, 0.45)\}$	$\{(s_4, 0.25), (s_7, 0.75)\}$
FM_5	$\{(s_1, 0.2), (s_2, 0.3), (s_7, 0.5)\}$	$\{(s_6, 0.3), (s_7, 0.4), (s_8, 0.3)\}$	$\{(s_1, 0.7), (s_4, 0.3)\}$
FM_6	$\{(s_6, 0.4), (s_7, 0.25), (s_8, 0.35)\}$	$\{(s_5, 0.65), (s_6, 0.35)\}$	$\{(s_6, 0.3), (s_7, 0.45), (s_8, 0.25)\}$

Using (9), we can obtain that

$$PV^{(1)} = (4.9038, 6.1125, 5.2075, 5.28, 5.35, 5.0625)^T,$$

$$PV^{(2)} = (5.03, 5.2375, 5.9375, 6.575, 6.1775, 4.875)^T,$$

$$PV^{(3)} = (5.1375, 6.1763, 4.8675, 5.1425, 6.9, 6.15)^T,$$

$$PV^{(c)} = (5.0127, 5.7784, 5.4145, 5.7636, 6.0685, 5.2594)^T.$$

Furthermore

$$R^{(1)} = (3, 1, 2, 2, 1, 3)^T, R^{(2)} = (3, 2, 2, 1, 1, 3)^T,$$

$$R^{(3)} = (3, 1, 3, 2, 1, 2)^T, R^{(c)} = (3, 1, 2, 2, 1, 3)^T.$$

Using (14), the consensus level can be obtained, that is, $cl_2 = 0.1111$. This indicates that the predefined consensus level among all FMEA members has been achieved.

According to $R^{(c)} = (3, 1, 2, 2, 1, 3)^T$, the collective ordinal classifications regarding the FMs are $C_1 = \{FM_2, FM_5\}$, $C_2 = \{FM_3, FM_4\}$, and $C_3 = \{FM_1, FM_6\}$. Therefore, the most important failure modes are FM_1 and FM_2 , which should be considered of great concern for risk mitigation.

VIII. COMPARISON ANALYSES

In this section, we compare our consensus-based FMEA method with existing FMEA methods [7], [9], [13], [29], [39]–[42], [55]. In particular, the most distinctive features of the proposed consensus-based FMEA method are identified and compared below against the main characteristics of nine related FMEA methods.

- FMs assessments:** The PHFLTS is a very effective decision-making tool owing to its convenience and flexibility in handling the hesitancy and uncertainty in practical contexts. In this study, the FMEA members are assumed to use PHFLTS to express their assessment information regarding the FMs with respect to the risk factors.

- Risk analysis results of FMs:** Almost all existing FMEA methods are focused on how to yield the complete ranking of FMs from the highest to the lowest risk. This study focuses on the ordinal classification of FMs owing to the

TABLE XVI
COMPARISONS OF THE CONSENSUS-BASED FMEA METHOD AND THE EXISTING FMEA METHODS

FMEA methods	FMEA context	FMs assessments	Method to determine weights of risk factors	Risk analysis results of FMs	Consensus
Certa et al. [9]	Individual	1-10 numerical points	Not considered	Ordinal classifications	Consensus issue does not exist owing to the fact that only one member is involved
Cantone et al. [7]	Group	1-10 numerical points	Not considered	Complete rankings	Not considered
Chin et al. [13]	Individual	1-10 numerical points	DEA model	Complete rankings	Consensus issue does not exist owing to the fact that only one member is involved
Huang et al. [29]	Group	Linguistic distribution assessments	Entropy-based integrated approach	Complete rankings	Not considered
Liu et al. [39]	Group	Interval two-tuple linguistic set	Integrated approach	Complete rankings	Not considered
Liu et al. [40]	Group	Hesitant two-tuple linguistic term sets	Multiple objective optimization model	Complete rankings	Not considered
Lolli et al. [41]	Group	1-10 numerical points	Not considered	Ordinal classifications	Not considered
Mohsen and Fereshteh [42]	Group	Z number	Shannon entropy	Complete rankings	Not considered
Wang et al. [55]	Group	Rough number	Subjective method	Complete rankings	Not considered
The proposed method	Group	PHFLTS	Consensus-driven approach	Consensual ordinal classifications	Optimization-based consensus model

fact that the complete ranking of FMs is sometimes very time-consuming and unnecessary. Moreover, the ordinal classification of FMs can provide a clear indication on which FMs are corrected first [41].

- 3) *Methods used to determine the weights of risk factors:* Although several approaches have determined the weights of risk factors, a method from a consensus perspective is still lacking. This study developed a consensus-driven optimization-based model to determine the weights of the risk factors.
- 4) *Consensus decision:* In this study, the consensus issue among FMEA members was addressed, and an optimization-based model with minimum information loss was constructed to support achievement of consensus. To the best of our knowledge, this is the first FMEA method that is capable of dealing with consensus issues over the course of the FMEA, thereby providing consensual collective decision results.

Moreover, the detailed comparisons between the existing FMEA methods and the consensus-based FMEA method are described in Table XVI.

IX. CONCLUSION

This study investigated the ordinal classification-based FMEA problem of FMs with the possibilistic hesitant fuzzy linguistic information, and developed a consensus-based MAGDM approach to obtain the ordinal risk classes of the FMs. In the proposed FMEA approach, the FMs were classified into several ordinal risk classes rather than into a complete risk order. Meanwhile, an optimization-based consensus model with the minimum adjustment distance was proposed to support achievement of consensus regarding the obtained collective ordinal risk classifications of FMs. This optimization-based consensus model was also transformed into a 0–1 mixed linear programming model. The feasibility and validity of the proposed consensus-based FMEA approach was justified using a case study regarding the risk analysis in proton beam radiotherapy. Moreover, the comparison analysis showed that our study constructed a

novel FMEA framework with several added values with respect to previous related approaches.

Meanwhile, three interesting and noteworthy directions for future research are pointed out.

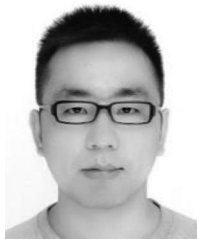
- 1) Recently, the analysis of social relationship information and determination of the weights of individuals based on social network analysis have emerged as a hot topic in GDM and MAGDM problems [56]. Therefore, we believe that it will be very interesting for future research to develop a social network-analysis-based framework for supporting the process of reaching consensus in FMEA problems.
- 2) Real-world FMEA problems involve not only mathematical aspects but also psychological behaviors of FMEA members. We argue that it will be interesting to investigate the psychological behaviors of FMEA members in the process of reaching consensus in FMEA problems.
- 3) To our knowledge, there is a lack of framework aimed at comparing different FMEA methods. Thus, it is necessary in future research to propose criteria to compare our proposal with other FMEA methods.

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